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# THE COMPUTATION OF FIRING TABLES FOR GUIDED MISSILES

by

Harold J. Breaux



November 1966

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U. S. ARMY MATERIEL COMMAND
BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

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# THE COMPUTATION OF FIRING TABLES FOR GUIDED MISSILES

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RDT & E Project No. 1P523801A287

ABERDEEN PROVING GROUND, MARYLAND

BALLISTIC RESEARCH LABORATORIES

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# THE COMPUTATION OF FIRING TABLES FOR GUIDED MISSILES

## ABSTRACT

The computation of firing tables for guided missiles is a problem that first arose when the U.S. Army introduced the Redstone Missile into its arsenal of weapons. The repetitive nature of such computations, their continuing requirement, and the turnover in personnel create a need for systemization of computations, applicable for different missile systems and at the same time, not requiring extensive analysis, new computer programs, and training of personnel. Recent work done by the Computing Laboratory of BRL accomplishes this objective and is a significant improvement in the state of the art. This report describes the procedure and includes a realistic example of a typical computational problem.

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TABLE OF CONTENTS

## LIST OF SYMBOLS

| Term | Definition |
|---|---|
| $A$ | Altitude of launcher |
| $A_a$ | Aiming azimuth |
| $A_t$ | Target azimuth |
| $D$ | Deflection of the missile from the aiming direction |
| $\vec{g}$ | Gravity vector |
| $\vec{F}$ | Vector representing the sum of propulsive, aerodynamic and gravitational forces |
| $g_{45}$ | Magnitude of gravity at 45° latitude |
| $H$ | Height of target above launcher |
| $L$ | Latitude of launcher |
| $R$ | Range between launcher and target |
| $T$ | Time presetting |
| $V$ | Presetting governing the attained range |
| $V(R, H, A)$ | Symbolism used to denote a function of the three variables R, H and A |
| V1 | Symbolic variable used in multiple regression |
| $\vec{x}_1, \vec{x}_2, \vec{x}_3$ | Triad of unit vectors specifying a non-inertial coordinate system fixed on the Earth's surface at the launch point |
| $\dot{X}_1, \dot{X}_2, \dot{X}_3$ | Velocity components in $\vec{x}_1, \vec{x}_2, \vec{x}_3$ direction |
| $\ddot{X}_1, \ddot{X}_2, \ddot{X}_3$ | Acceleration components in $\vec{x}_1, \vec{x}_2, \vec{x}_3$ direction |
| $\delta R$ | Range perturbation |
| $\Delta A$ | Target azimuth minus aiming azimuth $(A_t - A_a)$ |
| $\xi c$ | Dummy variable used in climatalogy |
| $\vec{\omega}$ | Earth's angular velocity vector |
| $\omega$ | Magnitude of $\vec{\omega}$ |

# INTRODUCTION

The computation of firing tables for guided missiles is a problem that first arose when the U.S. Army introduced the Redstone missile into its arsenal of weapons. The method of solution adopted for the Redstone became a model for the solution of similar problems on the Jupiter and Pershing missiles. This work, which was done at the Army Ballistic Missile Agency and at the U.S. Army Ballistic Research Laboratories, is documented in References 1 through 5*.

Primary responsibility for the computation of firing tables for guided missiles lies with the Commanding General of the Army Missile Command or with U.S. Army Materiel Command Project Managers. This responsibility is specified in AMC Regulation No. 310-9 dated 29 April 1963. This regulation allows the Army Missile Command to request the assistance of the Ballistic Research Laboratories or to include in the development contracts with industry, provisions for firing table computations. Because of artillery doctrinal procedures and the experience of government agencies as cited above, industry (when it has been involved) has turned to these agencies for technical guidance in the area of firing table computations. As a result, the existing techniques have been utilized by industry as well as by government agencies and is considered to be the state of the art.

The repetitive nature of such computations, their continuing requirement, and the turnover in personnel create a need for systemization of computations, (applicable for different missile systems and at the same time not requiring extensive analysis), new computer programs, and training of personnel.

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* *References may be found on page 49.*

Recent work done by the Computing Laboratory of BRL accomplishes this objective and is a significant improvement in the state of the art. The technique which has been devised, after its mechanization, makes the problem routine. The procedure has been used successfully in the development phase of a major U.S. Army guided missile system, and test exercises indicate that it will be applicable to guided missile systems in general.

The essential difference between the procedure to be described and that used previously is the use of stepwise multiple regression. The mathematical basis for the technique is an algorithm due originally to M.A. Efroymson[6*]. To fully understand the implementation of the computations described herein, the reader is referred to a report by the author and others[7].

*Superscript numbers denote references which may be found on page 49.

## NATURE OF THE PROBLEM

Despite the fact that a missile system may have "on board" guidance, a requirement may still exist for determining, prior to launch, the presettings required on the missile so that the missile's trajectory will pass through a desired target point. This is apparent from an examination of the factors which influence the missile's flight. These factors can be divided into two categories.

1. Controllable factors
   a. Physical Characteristics
   b. Propulsion
   c. Guidance and control electronics
   d. Launch conditions
   e. Fuzing
2. Uncontrollable factors
   a. Atmospheric conditions (meteorology)
   b. Effects of the Earth's rotation, gravitational field, and geometry.

Presettings are defined as those factors which are purposely varied or controlled so that a desired trajectory may be obtained. Those factors which are uncontrollable but produce predictable results on the trajectory lead to the concept of compensation, i.e., the presettings are adjusted to compensate for known effects on a trajectory caused by uncontrollable factors. Some uncontrollable factors have predictable results, but, because of the uncertainty of the factor itself, it may not be fruitful from an accuracy standpoint to correct for it or to consider it a variable of the problem. Such is often the case with meteorology as will be discussed in a later section.

The design and production objectives are to produce missiles of statistical similarity in areas 1a. 1b. and 1c. The degree to which these objectives are met will be a contributing factor to the accuracy of the system. The effects caused by meteorology create an additional design objective, namely, relative insensitivity to meteorological conditions.

The traditional presentation of presettings in tabular form for cannon and rocket artillery is called a firing table. In recent years gunnery procedures have, of necessity, become more sophisticated and as a result, more complicated. This has led to the introduction and use of field computers. Accordingly, the computational scheme and associated equations for the computation of presettings for guided missiles, though not necessarily utilizing the tabular presentation of classical firing tables, nevertheless retain the nomenclature, "Firing Tables".

Prior to and during the development and testing phase of a missile system's history, a concentrated effort is made to develop a mathematical model which represents the flight of the missile. This effort also includes an attempt to verify or determine the accuracy of the analytical representations of factors 1a, 1b. and 1c. Reasons for any significant discrepancies between the mathematical model and test flights are sought and corrected. This mathematical model assists in design, range safety, effectiveness studies, and system planning and is completely essential for firing table computations. The flight tests, by practical necessity are conducted at one or several locations. The number of such flight tests is small, usually only that required to completely determine the soundness of the design and engineering and production specifications and to collect sufficient data to verify the validity of the mathematical model. Since the missile, when fielded, should be capable of being fired from any point on the Earth, in any direction, and to any range within its capability, some method is required to predict the flight of the missile in situations different from the conditions and locations existing during the test phase. This capability is provided by the mathematical model since it can be used to generate trajectory data for any condition of the factors which influence the trajectory.

In employing his missile, the missileman is faced with the following problem. His missile is located at some point on Earth (usually specified by UTM coordinates) and he desires to engage a target at some other point, again specified perhaps by UTM coordinates. He must determine the launch conditions and presettings which, when inserted into the missile's intelligence, will cause the missile to fly a trajectory through the target point and burst the warhead at the appropriate time. To assist in the solution of this problem, the missileman is given a firing table.

## FACTORS WHICH INFLUENCE FIRING TABLE DESIGN

The mathematical model described earlier usually takes the form of a system of ordinary differential equations which govern the attitude and motion of the center of gravity of the missile. All seven factors which were mentioned in the previous section and which influence the flight of the missile, are represented in the model. The solution of the system of equations results in a theorectical trajectory.

Once the physical characteristics, propulsion, and guidance and control electronics have been fixed or standardized in the model, these factors are no longer variables in regard to solution of the gunnery problem. This is true only so long as the tactical missiles have these factors relatively unchanged from the standard. As a result, the theoretical trajectories are influenced by the launch conditions, meteorology, presettings, and the Earth model.

For the present, it will be assumed that the design of the missile has been such as to negate any significant effects of meteorology. Hence the meteorology is treated as a standard factor and accordingly is not a variable. Because the firing table problem is essentially mathematical, the remaining factors, launch conditions, presettings, and Earth model need to be represented by mathematical variables. The UTM coordinates mentioned above, are usually transformed into geographic coordinates by use of UTM-Geographic transformations*. These computations then yield range angle, latitude, and target azimuth.

If the launch conditions on a missile are fixed but the latitude of launch and aiming azimuth are varied, the range and deflection of the impact point will differ. This occurs because of the Coriolis and centrifugal accelerations imparted to the missile due to the Earth's rotation. In addition, the Earth's gravitational field** varies significantly with latitude due to its ellipsoidal nature. The shape produces additional geometrical effects. It follows that any scheme for determining presettings would have to take into account variations related to L, latitude of launch, and $A_t$, target azimuth. In addition, the presettings would be expected to vary with R, range between launcher and target, H, height of target above the launcher, and A, altitude of launcher.

The presettings are the dependent variables of the firing table problem, and the five variables (R, H, A, L, $A_t$) are the independent variables. The nature and total number of presettings vary from system to system. In general, however, all missile systems have three parameters which are in some way related to three basic presettings. One of these corresponds to a setting imposed at launch that is designed to achieve the desired range, such as quadrant elevation, engine cutoff time, accelerometer setting, velocity, etc. For the sake of generality,

*A UTM-Geographic transformation scheme adaptable for field computers can be found in Reference 8.

**Note that there exists a distinction between the terms "gravitation" and "gravity". Gravitation is the mutual attraction between masses of matter. Gravity is the vector sum of two opposing forces: Gravitation and the centrifugal force due to the rotation of the Earth. See Reference 9.

it will be denoted as V and could correspond to any of the above. Warhead fuzing requirements usually create a need for a second presetting, T, which is related to the time of flight or time of arrival at the target. The third common presetting is $A_a$, the aiming azimuth.

For the purpose of a typical firing table there arises a need for the specification of three presettings, each a function of five variables. These presettings, (functions in the firing table) are designated

$$
\begin{aligned}
V &= V(R, H, A, L, A_t) \\
T &= T(R, H, A, L, A_t) \qquad (1) \\
\Delta A &= A_t - A_a = \Delta A(R, H, A, L, A_t)
\end{aligned}
$$

The presentation of the formulas and any associated computations comprises the firing table. Ideally a firing table should possess two major characteristics, simplicity and accuracy. These two characteristics should provide a rapid solution to the gunnery problem without requiring complicated equipment to assist in computation. The characteristic of simplicity is usually met by the tabular or manual firing table in that it requires no computational aids beyond field-type hand crank calculators. For some systems, however, the tabular or manual firing table may require too much bulk, may not be sufficiently accurate, or may require excessive solution time. In these instances, the Army has turned to small field computers organic to the weapon system, as was the case in Redstone, Jupiter, Pershing and Sergeant or, more recently, to general purpose field computers such as FADAC.

One approach to solving the firing table problem would be to program the mathematical model as described earlier for the field computer. This has been done for cannon artillery systems and for the Little John and Honest John rockets. The practicality of this procedure is influenced by two major factors. The first of these is related to the degree of complexity of the mathematical model and possible simplifications. The second results from the fact that the independent variables of a trajectory do not correspond to the independent variables of the firing table problem. In the computation of a trajectory the presettings are first selected and then a trajectory is computed. The terminal point of the trajectory corresponds to some fictitious target. Hence, to make the mathematical model useful in this application, the process of iteration must be introduced, i.e., the presettings must be continually adjusted until the trajectory corresponds to the desired one. Usually this would require three or more trajectory computations, and, since a trajectory computation consists of the numerical integration of a system of differential equations, thousands of mathematical operations are involved. Generally, it is desirable to keep the solution time to less than five minutes; thus the programming of the mathematical model on the field computer may not be practical. The alternative is the determination of a simple collection of formulas for the presettings, represented symbolically by Equation (1), which can be readily evaluated on a field computer. It is toward this end that this report is directed.

## DEVELOPMENT OF THE METHOD

The most obvious approach to the obtaining of formulas such as Equation (1) is to resort to the established procedures of curve fitting. Two general categories seem applicable, Orthogonal Functions and Least Squares. The theory of orthogonal functions is developed for one variable and, since the problem being considered has five, some adaptation would become necessary. The most serious problem associated with least-squares is the usual one, namely, that one has to choose the form of the linear model beforehand and any subsequent change in the form of the model necessitates a recomputation of coefficients. Masaitis[5] points out that the assumption of a model containing three terms for each variable and all resulting combinations of the five variables creates a model of $3^5$ or 243 terms. Even if this model contained all the necessary terms, the computational problem of solving the resulting normal equations for such a large problem might be near impossible. Despite this seemingly overwhelming difficulty, the method to be presented herein will be essentially that, namely an attempt to choose a form of linear model which can be fitted directly in one operation. Prior to discussing this procedure, a brief examination of the existing technique is undertaken.

## TRUNCATED FOURIER SERIES TECHNIQUE

The technique used previously has often been referred to as the "Truncated Fourier Series Technique". The method actually employs least-squares extensively but has acquired this name because of the use of numerical harmonic analysis to determine the variation of the presettings with latitude and azimuth.

The technique begins with the assumption that the five variables $(R, H, A, L, A_t)$ can be divided into two groups based on the magnitude of their significance. These two groups are $(R, L, A_t)$ and $(H, A)$. In accord with the division of the variables into these two groups arises the concept of a sea level firing table, i.e., a firing table with $(H, A) = 0$, and "corrections" to the sea level firing table to account for nonzero values of $(H, A)$. The sea level formulas then take the form:

$$\begin{aligned} V &= V\,(R, L, A_t) \\ T &= T\,(R, L, A_t) \\ \Delta A &= \Delta A\,(R, L, A_t) \end{aligned} \qquad (2)$$

For a given range, R, and latitude, L, an intuitive argument can be made for the approximation of (V, T, $\Delta A$) in a Truncated Fourier Series in $A_t$. It follows that for fixed R and varying L the coefficients of the above series could be approximated by a truncated series in L. This leads to the technique of cross fitting. To approximate periodic data by a Fourier Series one can use the twelve ordinate method of numerical harmonic analysis[10]. In using this method, the data must be tabulated at 30° intervals.

At this point, the following problem has to be considered. The dependent and independent variables in the data analysis problem do not correspond to dependent and independent variables in the data gathering scheme, i.e., the generation of trajectories. For example, to compute (V, T, $\Delta A$) for fixed values of (R, L, $A_t$) requires iteration so that the data can be tabulated for $A_t$ = 0°, 30°, 60°, ..., 330° and for constant values of R. This iteration would be required for both variables $A_a$ and V. The computation of trajectories is usually quite expensive in regard to computer time; hence, the procedure is forced to take a different path. Instead of (2), equations of the following form are sought initially.

$$\begin{aligned} R &= R(A_a) \\ T &= T(A_a) \\ \Delta A &= \Delta A(A_a) \end{aligned} \tag{3}$$

The coefficients of the Truncated Fourier Series for functions such as (3) are determined for various values of V and for L = 0°, 30°, 60°, 90°. The resulting approximations, (3), are then used to retabulate the data as functions of $A_t$. The data are needed for $A_t$ = 0°, 30°, 60°, ..., 330° because of the requirements of the twelve ordinate scheme. This is accomplished in the following fashion. In the third equation of (3), $\Delta A = A_t - A_a$. In this equation $A_t$ is set to 0°, 30°, ..., 330° and

$A_a$ is obtained by Newton iteration. The resulting values of $A_a$ are then inserted into the first two equations of (3) and R and T are evaluated. The resulting data for (R, T, $\Delta A$) can then be refitted as functions of $A_t$. Equations of the following form result for L = 0°, 30°, 60°, 90° and for various values of V:

$$
\begin{aligned}
R &= R(A_t) \\
T &= T(A_t) \qquad (4) \\
\Delta A &= \Delta A(A_t).
\end{aligned}
$$

The coefficients of (4) are then ready to be cross-fitted in a Truncated Fourier Series in L. Data for L = 120°, 150°, ..., 330°* are obtained by sinusoidal extrapolation. The cross-fitting is done for each value of V. Finally, the coefficients of the truncated series in (L, $A_t$) are approximated by least-squares polynomials in V. One variant of the method, as used by Masaitis on the Pershing system[5], consists of fitting by least-squares the linear model containing the terms in (L, $A_t$) found to be significant in the procedure just described. This is done prior to cross-fitting the coefficients in terms of a series in V. In this way, the errors occurring in the inversion of Equation (3) and those attendant to the approximations of the twelve-ordinate method can be eliminated, and the resulting fit has the minimum variance for the terms that are in the approximation.

In a stepwise manner somewhat related to the above procedure, but much less straightforward, the corrections to the sea-level firing table are obtained. The end result is a collection of formulas of the type

$$
\begin{aligned}
R &= R_1(V, L, A_t) + R_2(V, L, A_t, H, A) \\
T &= T_1(V, L, A_t) + T_2(V, L, A_t, H, A) \qquad (5) \\
\Delta A &= \Delta A_1(V, L, A_t) + \Delta A_2(V, L, A_t, H, A)
\end{aligned}
$$

*Physically L has meaning only in the interval, $-90° \le L \le 90°$.

In the field, the firing table problem is as follows. Given targeting data (R, L, $A_t$, H, A), what values of (V, T, $\Delta A$) are required to engage the target? This implies that the first equation of (5) must be solved by iteration. While not a very difficult computation to perform on a field computer, this can create problems if one wants to adapt formulas such as (5) to manual solutions.

## MULTIPLE REGRESSION TECHNIQUE

The method just described, though quite ingenious, arises or is made necessary because of shortcomings or difficulties associated with more direct methods. The "Truncated Fourier Series Technique", as presently used, is really an art and has consistently required the close involvement of highly skilled individuals. Furthermore, the technique is not too easily adaptable for guided missile systems of short range. In these systems, the target height variable, H, and , in some instances, even A produce greater variations in the presettings than (L, $A_t$). As will be shown later for these systems, the form of the variations caused by (L, $A_t$) is well known, and the problem of fitting is more severe in the other three variables.

The previously mentioned problem associated with least-squares was largely eliminated by a method first employed by M. A. Efroymson[6]. Efroymson showed that the solution of the normal equations by the Gauss-Jordan algorithm can be made equivalent to solving a series of least-squares problems or models each differing from the previous problem by the inclusion of an additional term in the linear model. In this method the advantages associated with the use of orthogonal functions are retained, i. e., the labor associated with computing a number of coefficients is not lost when additional terms are introduced into the approximation. The procedure is controlled by use of correlation theory, which allows the flow of computations to be so directed that the most significant terms of a "candidate" linear model are entered into a "reduced" linear model which is being sought as the solution of a data analysis problem such as the one being considered here. A general outline of the computational aspects of this scheme can be found in Reference 6. The statistical theory underlying the control aspects of this scheme can be found in Reference 7 along with a general description of an existing computer program. This program allows one to assume extremely large linear models (100-150 terms) in fitting data without creating the usual problems of inverting matrices of high order. Very large matrices are formed initially, but the algorithm is controlled so as to terminate the solution before the inclusion of insignificant terms. The progression of the approximation can be stopped on a basis of statistical significance tests or when a prescribed standard deviation of residuals is achieved. The final solution normally would contain 20 to 50 terms, and in the process of solution those matrix elements which are unique to the smaller problem are not affected by the consideration of all the other terms. The computational errors which would arise in solving the complete problem (i. e., the entire candidate model) are not encountered and the solution is identical to that which would be obtained by fitting directly the smaller model by the conventional least-squares method. The conventional method, however, gives no clues as to how to define the smaller model, a shortcoming which has necessitated the cross fitting approach just described.

## CHOICE OF LINEAR MODELS

Any attempt to write a general formula, which might include all the terms necessary to adequately represent a collection of firing table data, would of necessity include many terms. The total number of terms can be made manageable by examining, in a qualitative manner, the factors which cause the variation of the presettings. The most fruitful area is the examination of the effects of latitude and azimuth.

The effects of $(L, A_t)$ due to the rotation of the earth, the variation of the gravitational field with latitude, and the geometrical effects caused by the ellipsoidal nature of the earth, influence the flight of a missile and therefore the presettings. In the simulation of theoretical trajectories these effects are incorporated by use of mathematical models. The form of these models differ, but all produce essentially the same results as obviously must be the case because all forms are intended to represent the same physical problem. The effects of the rotation of the earth are represented in some trajectory programs by imparting to the missile in inertial space and initial velocity equivalent to the linear velocity of the launch point on the earth's surface. The position of the earth relative to inertial space is maintained in the program, and the position of the missile relative to the earth is determined by use of geographical coordinates (latitude and longitude). This type of simulation serves to represent the influence of the psuedo accelerations mentioned previously plus any additional effects due to geometry. In this type model, the gravitational field is usually simulated by approximations in the form of spherical harmonic functions.

The effects of $(L, A_t)$, however, can be studied more easily by considering the equations of motion in noninertial space, i.e., relative to a coordinate system attached to the rotating earth. The effects being considered affect mainly the motion of the missile's center of gravity and, for the intended purpose, the three-degree-of-freedom equations suffice. If a missile or projectile is represented by a point mass and fired on a nonrotating earth, its motion can be represented by the vector differential equation.

$$m \ddot{\vec{X}} = \vec{F}. \qquad (6)$$

$\vec{X}$ is the missile's position, m is its mass and $\vec{F}$ is the vector sum of propulsive, aerodynamic and gravitational forces. This equation arises from Newton's second law and holds only in inertial space. To make Equation (6) valid for trajectories computed on a rotating earth, it is customary to introduce into (6) the psuedo accelerations mentioned above. The Coriolis acceleration can be represented by the vector, $(-2\,\vec{\omega} \times \dot{\vec{X}})$ and the centrifugal acceleration by the vector, $\vec{\omega} \times (\vec{\omega} \times \vec{R})$ where $\times$ is the vector cross product, $\vec{\omega}$ is the angular velocity of the earth, $\dot{\vec{X}}$ is the missile's velocity, and $\vec{R}$ is the vector from the earth's center to the missile. Prior to being launched, a missile is elevated relative to a reference determined by the local direction of gravity (the plumb bob). The gravity vector, yielding this reference, consists of the gravitation and the local centrifugal acceleration. This consideration, when ignored, produces erroneous results when the centrifugal acceleration is introduced as $\vec{\omega} \times (\vec{\omega} \times \vec{R})$. A downrange component of centrifugal acceleration enters unless the coordinate system is tilted with respect to the mathematical normal to the reference ellipsoid. This problem is easily avoided by writing the equations of motion in the form.

$$\ddot{\vec{X}} = \vec{F}/m + \vec{g} - 2\,\vec{\omega} \times \dot{\vec{X}} \qquad (7)$$

$\vec{g}$ is defined as the gravity acceleration which includes the effects of gravitation and centrifugal acceleration. The coordinate system has $\vec{x}_1$ pointed in the direction of fire, $\vec{x}_2$ along the plumb line and pointing upward, and $\vec{x}_3$ completes the triad. In this coordinate system $\vec{\omega}$ takes the form

$$\begin{aligned} \vec{\omega} = \omega \cos L \cos A_a\, \vec{x}_1 + \omega \sin L\, \vec{x}_2 \\ -\omega \cos L \sin A_a\, \vec{x}_3 . \end{aligned} \qquad (8)$$

As shown in Reference 6 the magnitude of $\vec{g}$ can be closely approximated by

$$\left|\vec{g}\right| = g_{45} \quad (1 - \gamma \cos 2L) \,. \tag{9}$$

After the insertion of (8) and (9) into (7) and the introduction of an altitude dependency into $\vec{g}$, (7), as shown in Reference 11, can be approximated by

$$\begin{pmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \end{pmatrix} = \begin{pmatrix} F_1/m - g_{45} \; X_1/R \\ F_2/m - 2g_{45} \, X_2/R \\ F_3/m - g_{45} \, X_3/R \end{pmatrix} + \tag{10}$$

$$\begin{pmatrix} g_{45} \; \gamma \; \cos 2L \; X_1/R - 2\omega \cos L \sin A_a \; \dot{X}_2 - 2\omega \sin L \; \dot{X}_3 \\ g_{45} \; \gamma \; \cos 2L \; (1 - 2X_2/R) + 2\omega \cos L \sin A_a \; \dot{X}_1 \\ \qquad + 2\omega \cos L \cos A_a \; \dot{X}_3 \\ g_{45} \; \gamma \; \cos 2L \; X_3/R + 2\omega \sin L \; \dot{X}_1 - 2\omega \cos L \cos A_a \; \dot{X}_2 \end{pmatrix}$$

After exhibiting the equations of motion in the form (10), one can take advantage of the theory of differential variations, as developed by Moulton[10], to find the effects of ($L$, $A_t$) on range, time of flight, and azimuth deflections. The second matrix on the right in (10) consists of minor factors which are quite small in comparison to the total accelerations. These differential accelerations lead to the concept of differential corrections. Standard trajectories are defined as those computed with the matrix of minor factors set to zero. For standard trajectories and fixed values of the presettings, launch altitude, and target height, the range, time of flight , and linear deflection from the launch azimuth are denoted respectively by $R_0$, $T_0$, $D_0$. In the presence of the minor factors, the theory of differential variations leads one to expect the range, time of flight, and deflection to take the form:

$$R = R_0 + R_1 \cos 2L + R_2 \cos L \sin A_a$$

$$T = T_0 + T_1 \cos 2L + T_2 \cos L \sin A_a \qquad (11)$$

$$D = D_0 + D_1 \sin L + D_2 \cos L \cos A_a$$

$R_1$, $R_2$, $T_1$, and $T_2$ are small compared to $R_0$ and $T_0$ and are called differential variations. $D_0$ is present usually for spinning missiles and is commonly called drift. $D_0$, $D_1$ and $D_2$ are also small. All of the minor factors which included $X_3$ and $\dot{X}_3$ were ignored because of their relative insignificance. The first equation of (11) can be written in the form

$$R = R_0 + \frac{\partial R}{\partial \alpha_1} \delta\alpha_1 + \frac{\partial R}{\partial \alpha_2} \delta\alpha_2 = R_0 + \delta R. \qquad (12)$$

$\delta R$ is a range perturbation equal to the sum of the component range perturbations, $\frac{\partial R}{\partial \alpha_1} \delta\alpha_1$ and $\frac{\partial R}{\partial \alpha_2} \delta\alpha_2$. Since $\delta R$ is small in comparison to $R_0$, an assumption which usually holds true in practice, and if $V_0$ and $V$ are the presettings corresponding to $R_0$ and $R$, respectively, the presetting $V$ can be approximated by the equation

$$V = V_0 + \frac{\partial V}{\partial R} \Big|_{R_0} \delta R. \qquad (13)$$

Since the range differential, $\delta R$, is caused by the minor factors,

$$\delta R = \frac{\partial R}{\partial \alpha_1} \delta\alpha_1 + \frac{\partial R}{\partial \alpha_2} \delta\alpha_2 = R_1 \cos 2L$$

$$+ R_2 \cos L \sin A_a$$

and

$$V = V_0 + \frac{\partial V}{\partial R} (R_1 \cos 2L + R_2 \cos L \sin A_a)$$

or

$$V = V_0 + V_1 \cos 2L + V_2 \cos L \sin A_a . \qquad (14)$$

If data were generated for different values of V, H and A one could obtain a collection of formulas such as (14). The collection could then be approximated by the formula

$$V = V_0(R, H, A) + V_1 (R, H, A) \cos 2L + V_2 (R, H, A) \cos L \sin A_a \qquad (15)$$

which is the same as (14) except that $V_0$, $V_1$ and $V_2$ are now to be considered as functions of (R, H, A).

$A_a$ is a dependent variable of the firing table problem and is not available until a formula similar to (15) is evaluated for $\Delta A$. $A_t$ and $A_a$ usually differ only slightly, and, since we are merely seeking the form of a candidate linear model, no problem arises if $A_a$ is replaced by $A_t$ in (15). The quantity $\Delta$ A is expected to behave similarly to the deflection D, and an equation similar to (15) can be expected for T. Hence, one is led to a choice of linear models for the presettings which take the following form:

$$V = V_0 (R, H, A) + V_1 (R, H, A) \cos 2L + V_2 (R, H, A) \cos L \sin A_t \qquad (16)$$

$$T = T_0 (R, H, A) + T_1 (R, H, A) \cos 2L + T_2 (R, H, A) \cos L \sin A_t \qquad (17)$$

$$\Delta A = \Delta A_0 (R, H, A) + \Delta A_1 (R, H, A) \sin L + \Delta A_2 (R, H, A) \cos L \cos A_t \qquad (18)$$

Experience indicates that the quantities $V_0$, $V_1$, etc. can be closely approximated by polynomials in (R, H, A). The question arises whether the formulas (16), (17), and (18) are sufficiently general in the variables (L, $A_t$). In practice it turns out that, for missile systems of short range formulas such as these are quite adequate. At long ranges the inclusion of additional trigonometric terms in (L, $A_t$)

are required. It is recognized that in this context the words short and long give the firing table planner nothing to work with and indeed, if the formulas are inadequate, how does one proceed to determine adequate formulas? It should be remembered that the purpose of this exercise was merely to establish a physical reasoning to gain insight into the formation of a candidate model. In any such problem, it is necessary to verify the adequacy of the approximations because of the differences that exist between missile systems. The process of verification serves as a safety check and throws light on the validity or falsity of any assumptions that are made.

Prior to considering the more general problem, the following significant point should be observed. Suppose that it was known beforehand that the only trigonometric terms that influence the presettings are those listed in the above formulas. This would be very significant in two areas:

(1) The size of the candidate model.

(2) The total amount of data required.

It was pointed out earlier that the formation of a model in five variables containing three terms in each variable and considering all combinations would result in 243 terms. The model based on formulas (16), (17) and (18) including all terms in R, H and A through the quadratic terms and all combinations would contain only $3 \times 3^3$ or 81 terms.

The data required to find approximations such as (16), (17) and (18) are far less than would be required in the more general case. For a sea level firing table the Truncated Fourier Series Technique might require as many as 30 trajectories for each value of the presettings as compared to as little as 3 for the technique being described. This creates a very strong motivation for the determination of the significant trigonometric terms prior to making decisions on how many and what type trajectories are to be run for firing table purposes.

No implication should be drawn from the previous discussion that trajectories should be computed on the basis of defining standard trajectories, or that three-degree-of-freedom kinematics should be used in generating the trajectory data. It is assumed throughout that the data would be generated by a six-degree -of-freedom trajectory model when required. The discussion was conducted merely to lend insight into the formation of candidate linear models.

## DETERMINATION OF SIGNIFICANT HARMONICS

The trigonometric harmonics in $(L, A_t)$ which affect the presettings significantly will be the same harmonics which affect the range, time of flight, and deflection in a significant fashion. Furthermore, those harmonics which have no significant affect on the missiles trajectory at the maximum range of interest are not apt to have any effect at ranges less than the maximum. These two statements are not rigorously true, but, nevertheless, seem plausible from the following physical reasoning. The trigonometric harmonics influence the equations of motion as differential accelerations and through the fact that the geometrical reference of the trajectory is with respect to a nonspherical Earth. The first of these factors causes differential displacements of the center of gravity which grows as the square of the time of flight. The second factor produces displacements which increase with the distance of the missile from the launch point. Hence, both factors produce effects which increase with range.

The fact that the range can be expressed as a linear combination of certain trigonometric terms does not guarantee that the presetting V can be approximated by a linear combination of the same trigonometric terms. This is so only if Equation (13) is sufficiently accurate to approximate V for the largest values of $\delta R$ that are possible. It must be recognized, however, that a problem of this type is not amenable to rigorous analysis, and , in this situation the following precept is a good one to follow. Establish from physical principles a plausible argument. From the intuition so gained, define a candidate model which seems to have a chance for success. Try it. If the results so obtained are sufficiently accurate, the job is done. If the results are not sufficiently accurate, re-examine the assumptions and try again.

The following type of analysis should suffice in determining the significant harmonics. For fixed presettings corresponding to maximum range and (H, A) = 0, one should compute trajectories for various latitudes, e.g., 0°, 15°, 30°, ..., 60° and at each latitude several values of $A_a$, e.g., 0°, 30°, 60°, ..., 330°. At the pole only one data point is required. From these data one should fit functions of the following form:

$$R : R(L, A_t)$$
$$T = T(L, A_t)$$
$$\Delta A = \Delta A(L, A_t)$$

This fitting is done quite simply by the Stepwise Multiple Regression procedure. The regression should be terminated after a preselected standard deviation of residuals is obtained. The reduced linear model then indicates the significant harmonics. As a candidate model one can choose all combinations of the sine and cosine harmonics in $(L, A_t)$ through the fifth harmonic and all possible products. An examination of fitted results for previous work on Redstone, Pershing, etc. indicate that this model should be sufficient.

Mention was made earlier with respect to developing a procedure which would take only one step. The procedure, as outlined thus far, implies two steps:

(1) Determination of significant harmonic terms and generating of trajectory data

(2) Fitting of candidate linear model.

The requirement for the first step, however, arises from consideration of economy and is not a requirement peculiar to the fitting technique. The importance of keeping economy under consideration in this problem is attested to by the following case history. This case history occurred during the preliminary planning for firing table computation on a guided missile system for which industry had contract responsibility. Not being experienced in this area, industry sought technical guidance from the Government, and, after mutual agreement, a plan of attack was agreed upon. This resulted in a "statement of intent" and included a general outline of a procedure similar to the Truncated Fourier Series Technique described earlier. The significant item in this plan for the sea-level firing table was the requirement for 61 trajectories for each value of the presettings.

These trajectories corresponded to the following values on L and $A_a$.

| L | $A_a$ |
|---|---|
| 0° | 0, 30°, 60°, ..., 330° |
| ± 30° | 0, 30°, 60°, ..., 330° |
| ± 60° | 0, 30°, 60°, ..., 330° |
| 90° | 180° |

This large number arose as a result of suspected asymmetries in the Northern and Southern Hemispheres. Experience since then indicates that anywhere from 50 to 80 values of the presettings would be required resulting in more than 3050 trajectories. The trajectories under consideration required about 15 minutes running time on an IBM 7090 computer. The total cost for just the sea level firing table would have exceeded 760 hours of IBM 7090 time. Even if funds were available, such an approach would not be practical because of considerations of reliability of data generated over such a time span and the availability of a computer. Subsequent analysis by BRL indicated that the only significant harmonics were those discussed to this point and that only three different conditions of $(L, A_a)$ were required to fit the data to the desired accuracy. The end result is a cost comparison of 760 hours versus 40 hours or a total difference of 95 percent. The rental on the IBM 7090 is usually greater than $500 per hour. This serves to illustrate the requirement for a thorough analysis of the required harmonic terms prior to generating very expensive data. This analysis provides very useful information in helping to choose a candidate linear model which can be fitted in one operation.

Some missile systems have guidance and control electronics which may be influenced directly by such things as the local magnitude of gravity. For example, a fixed value of a presetting inserted into the missile at different latitudes may produce different effective results on the hardware which vary with latitude and altitude. When this occurs, it is desirable, if possible, to redefine an intermediate variable which does not have this dependence. This intermediate variable replaces the basic presetting in the fitted approximations. The actual presetting is then computed as a function of the intermediate variable and a known dependence on the appropriate variables such as latitude and altitude. This type of procedure serves to keep the fitted approximations to the simplest possible form.

## SAMPLE PROBLEM

The sample problem presented below is a very realistic one. The mathematical model used to generate this data does not correspond to any missile either in use or in the planning stage but nevertheless, the data generated from this model have realistic variations in the presettings as a function of the five variables $(R, H, A, L, A_t)$. The computations are performed with respect to an inertial coordinate system at the Earth's center, the Earth is assumed to be the standard geoid of revolution, and the gravitational field is represented by the standard spherical harmonic functions. The range coverage is approximately 50 to 150 km. The data is too voluminous to tabulate here but is described in the table shown below.

DATA FOR STEP 1

(DETERMINATION OF SIGNIFICANT HARMONICS)

$L = 0°, 15°, 30°, 45°, 60°, 75°, \quad A_a = 0°, 30°, 60°, \ldots, 330°$

$L = \pm 90°, \; A_a = 180°$

$V = 1200, \; H = 0, \quad A = 0$

The data for the Southern Hemisphere are not really required but were included for the purpose of illustration. The stepwise multiple regression was performed on data for both Hemispheres and then for the Northern Hemisphere only. After the reduced model was obtained, it was refitted using five and then three data points. The results, as shown below, were comparable.

HARMONIC FITS

| | | Rms. Error |
|---|---|---|
| (1) | $R = 147831.16 + 413.39297 \cos 2L + 868.62314 \cos L \sin A_t$ | 4.0 M |
| (2) | $R = 147831.86 + 412.99783 \cos 2L + 868.99637 \cos L \sin A_t$ | 4.3 M |
| (3) | $R = 147830.96 + 414.14966 \cos 2L + 866.23731 \cos L \sin A_t$ | |
| (4) | $R = 147825.85 + 406.31939 \cos 2L + 879.45828 \cos L \sin A_t$ | |

## HARMONIC FITS

Rms Error

(1) $T = 197.39104 + .55285215 \cos 2L + 2.2659327 \cos L \sin A_t$ .0084 sec

(2) $T = 197.38903 + .55420584 \cos 2L + 2.2668419 \cos L \sin A_t$ .0064 sec

(3) $T = 197.38597 + .55773893 \cos 2L + 2.2699079 \cos L \sin A_t$

(4) $T = 197.37738 + .54257590 \cos 2L + 2.2969614 \cos L \sin A_t$

(1) $(A_t - A_a) = -3.4244540 + 14.495272 \sin L - 5.84783554 \cos L \cos A_t$ .03 Mils

(2) $(A_t - A_a) = -3.4135851 + 14.481395 \sin L - 5.8479165 \cos L \cos A_t$ .04 Mils

(3) $(A_t - A_a) = -3.4227257 + 14.504357 \sin L - 5.8482707 \cos L \cos A_t$

(4) $(A_t - A_a) = -3.4370314 + 14.462231 \sin L - 5.7664911 \cos L \cos A_t$

Code (1) - Both Hemispheres, (2) - Northern Hemisphere,
(3) - 5 Data Points, (4) - 3 Data Points

5 Data Points - $(L, A_a)$ = (0°, 0°), (0°, 90°) (45°, 0°) (45°, 90°), (90°, 180°)

3 Data Points - $(L, A_a)$ = (0°, 90°), (45°, 0°), (90°, 180°)

## DATA FOR STEP 2

(Fitting of Presettings)

$V = 650, 700, 750, \ldots, 1200$ meters/sec

$A = 0, 1000, 2000, 3000$ meters

$H = 0, 1000, 2000, 3000, 4000, 5000$ meters

$(L, A_a) = (0°, 0°), (0°, 90°), (45°, 0°), (45°, 90°), (90°, 180°)$

## CANDIDATE LINEAR MODEL

The candidate linear models for the sample problem were chosen after the results of Step 1 were obtained. The significant harmonic terms as indicated by the stepwise regression procedure were ($\cos 2L$, $\cos L \sin A_t$) for range and time and ($\sin L$, $\cos L \cos A_t$) for $(A_t - A_a)$. These were the same terms that were predicted to be significant from the analysis of the effects of $(L, A_t)$ on the three-degree-of-freedom equations. The fitting error, as noted above, was found to be small enough so that the inclusion of higher harmonic terms was not required for the range interval and data being considered. It should be pointed out that higher harmonic terms are to be expected as the range increases. This is evidenced by their presence in the corresponding work on Redstone and Pershing. This is of no consequence, however, in relation to the flexibility or generality of the procedure being described since the regression would have identified the appropriate higher harmonics in the order of their significance. An example with higher harmonic terms for data taken from Reference 2 is included in a later section.

The candidate linear model chosen for this example was limited to including less than one hundred terms because of a limitation in the program described in Reference 7. Such a limitation is made necessary because of memory storage capacity. This program, however, could be altered so that candidate models of 150 terms could be used if a requirement existed. The model is written in a form designed for the program and is listed exactly as input to the program. To understand the form of the model, the following rules are observed. The original data are assumed to be arranged sequentially on each line of input as V1, V2, V3, ... . The Vi's are symbolic variables accepted by the program to identify the data corresponding to the linear model. Any Vi can be fit as a linear combination of other Vi's or functions of other Vi's. The Ri variables, i = 1, 2, 3, ..., are called redefinition variables and serve to transform the original data. Transformations are required to guarantee that no intermediate numbers are generated of a magnitude which might exceed the machine capacity and to obtain computational accuracy. Furthermore, the field computers such as FADAC may have limitations on the allowable size of the coefficients themselves, and, hence, transformations serve to maintain small powers in the numbers. An effective technique is to transform the data by a linear transformation so that the spread of the data is transformed to the interval -1 to +1. Such a transformation is represented by the equation.

$$Ri = [2Vi - (Vi\ max + Vi\ min)]/(V_i\ max - V_i\ min),$$

where Vi max is the largest value of the Vi variable and Vi min the smallest value. The redefinitions can also be used to form initially the powers of the polynomial components of the model. This serves to minimize the computations related to forming the matrix elements and also simplifies the writing of the model. The data for this example, as related to the Vi symbolism, are

$$V1 = R\ (\text{meters})$$
$$V2 = (A_t - A_a)\ (\text{mils})$$
$$V3 = V$$

$V4 = H$ (meters)

$V5 = A$ (meters)

$V6 = L$ (deg)

$V7 = A_a$ (mils).

The variables V1, V2, V3, V4, and V5 are normalized to the interval (-1, 1), V6 is converted to mils by the formula R6 = 160* V6/9, and the quantity $A_t$ is obtained by adding V2 and V7. In addition, the powers of R, H, and A are formed. The following symbols are used.

| | |
|---|---|
| * | Multiplication |
| ** | Exponentiation |
| + | Addition |
| - | Subtraction |
| / | Division |
| S ( ) | Sine of enclosed argument |
| C ( ) | Cosine of enclosed argument |

The reasoning used in the formation of the candidate model for V is the following. The model is to take the general form of Equation (16). From experience and from observation of the data, it is known that the largest variation of V will be in the contribution of $V_0$. Hence, with a limitation of 100 terms, it is judicious to allot more terms to $V_0$ than to $V_1$ or $V_2$. The largest variation of $V_0$, $V_1$, and $V_2$ will be due to their dependence on R, and, hence, it is judicious to be generous in putting relatively more terms with R than with (H, A). Furthermore, it is suspected that H will be more significant than A. Within this general qualitative picture, any number of plausible candidate models could be chosen. The one listed in the sample problem arose out of the author's experience and is in no way put forth as the wisest choice. The power of the procedure lies in the fact that, with a minimum of intuition, one can usually find a good candidate either by choosing a very large model or by experimentation.

The input and output of the sample problem are listed on pages 38 through 41. The actual model for R3 which is the normalized V, begins with the statement, R3 = R21 + ... . Each term between the + signs is assumed to have a regression coefficient attached. Only the models and results for V and $(A_t - A_a)$ are listed. The candidate model for T was identical to V, and the results were comparable. The output solution model lists the sequence of terms as they are added to the regression and the corresponding progression of the standard deviation of residuals (CURR. ERMS). The actual solution consists of the various transformations followed by the listing of all the terms that were finally included in the regression and the corresponding coefficients. The regression was terminated when the standard deviation of residuals of V was less than .045 meters per second and of $(A_t - A_a)$ less than .07 mils. See BRL Report No. 1330 for a more detailed listing of input-output. The accuracy of the fits was checked by randomly generating 20 fire problems. The fits were evaluated, and this resulted in a collection of presettings. Theoretical trajectories were computed corresponding to these values of the presettings. The resulting data provided an estimate of the error of the fits. The 1$\sigma$ range error was 13 meters and the 1$\sigma$ cross track deflection error was 8 meters, both very acceptable magnitudes.

CANDIDATE LINEAR MODEL
VELOCITY

```
AH=2
NO RESIDUALS
STOP ERMS=.045
R1=(2*V1-190000)/110000,R3=(V3-925)/225,R4=(V4-2500)/2500,
R5=(V5-1500)/1500,R6=160*V6/9,R7=(V2+V7),R11=C(R6),R12=C(2*R6),
R14=S(R7),R16=R11*R14,
R21=R1,R22=R21*R21,R23=R21*R22,R24=R22**2,R25=R22*R23,R26=R23**2,
R31=R4,R32=R31**2,R33=R31*R32,R34=R32**2,
R41=R5,R42=R41**2,
R3=R21+R22+R23+R24+R25+R26
+R31+R31*R21+R31*R22+R31*R23+R31*R24+R31*R25+R31*R26
+R32+R32*R21+R32*R22+R32*R23+R32*R24+R32*R25+R32*R26
+R33+R33*R21+R33*R22+R33*R23+R33*R24+R33*R25+R33*R26
+R34+R34*R21+R34*R22+R34*R23+R34*R24+R34*R25+R34*R26
+R41+R41*R21+R41*R22+R41*R23+R41*R24
+R42+R42*R21+R42*R22+R42*R23+R42*R24
+R41*R31+R41*R31*R21+R41*R31*R22+R41*R31*R23+R41*R31*R24
+R42*R31+R42*R31*R21+R42*R31*R22+R42*R31*R23+R42*R31*R24
+R41*R32+R41*R32*R21+R41*R32*R22+R41*R32*R23+R41*R32*R24
+R42*R32+R42*R32*R21+R42*R32*R22+R42*R32*R23+R42*R32*R24
+R12+R21*R12+R22*R12+R23*R12+R24*R12
+R16+R21*R16+R22*R16+R23*R16+R24*R16
+R31*R12+R31*R21*R12+R31*R22*R12+R31*R23*R12
+R31*R16+R31*R21*R16+R31*R22*R16+R31*R23*R16
+R32*R12+R32*R21*R12+R32*R22*R12
+R32*R16+R32*R21*R16+R32*R22*R16
+R41*R12+R41*R21*R12+R41*R31*R12+R41*R31*R21*R12
+R41*R16+R41*R21*R16+R41*R31*R16+R41*R31*R21*R16%
```

| RANGE METERS | TA-AA MILS | VELOCITY METERS/SEC | HEIGHT OF TARGET | ALTITUDE OF LNCHR | LATITUDE DEGREES | AIM AZ MILS | |
|---|---|---|---|---|---|---|---|
| 144876.022- | 3.3702 | 1200.0000 | 5000.0000 | .0000 | .0000 | 1600.0000 | FTTST10101 |
| 145750.381- | 3.3881 | 1200.0000 | 4000.0000 | .0000 | .0000 | 1600.0000 | FTTST10101 |
| 146612.149- | 3.4057 | 1200.0000 | 3000.0000 | .0000 | .0000 | 1600.0000 | FTTST10101 |
| 147460.433- | 3.4230 | 1200.0000 | 2000.0000 | .0000 | .0000 | 1600.0000 | FTTST10101 |
| 148294.089- | 3.4400 | 1200.0000 | 1000.0000 | .0000 | .0000 | 1600.0000 | FTTST10101 |
| 149111.641- | 3.4567 | 1200.0000 | .0000 | .0000 | .0000 | 1600.0000 | FTTST10101 |
| 132674.819- | 3.2533 | 1150.0000 | 5000.0000 | .0000 | .0000 | 1600.0000 | FTTST10201 |
| 133558.703- | 3.2721 | 1150.0000 | 4000.0000 | .0000 | .0000 | 1600.0000 | FTTST10201 |
| 134430.141- | 3.2906 | 1150.0000 | 3000.0000 | .0000 | .0000 | 1600.0000 | FTTST10201 |
| 135288.250- | 3.3087 | 1150.0000 | 2000.0000 | .0000 | .0000 | 1600.0000 | FTTST10201 |
| 136131.894- | 3.3266 | 1150.0000 | 1000.0000 | .0000 | .0000 | 1600.0000 | FTTST10201 |
| 136959.608- | 3.3440 | 1150.0000 | .0000 | .0000 | .0000 | 1600.0000 | FTTST10201 |
| 121000.143- | 3.1493 | 1100.0000 | 5000.0000 | .0000 | .0000 | 1600.0000 | FTTST10301 |
| 121893.220- | 3.1690 | 1100.0000 | 4000.0000 | .0000 | .0000 | 1600.0000 | FTTST10301 |
| 122773.562- | 3.1884 | 1100.0000 | 3000.0000 | .0000 | .0000 | 1600.0000 | FTTST10301 |
| 123640.764- | 3.2074 | 1100.0000 | 2000.0000 | .0000 | .0000 | 1600.0000 | FTTST10301 |
| 124493.710- | 3.2261 | 1100.0000 | 1000.0000 | .0000 | .0000 | 1600.0000 | FTTST10301 |
| 125330.953- | 3.2444 | 1100.0000 | .0000 | .0000 | .0000 | 1600.0000 | FTTST10301 |
| 109844.303- | 2.9947 | 1050.0000 | 5000.0000 | .0000 | .0000 | 1600.0000 | FTTST10401 |
| 110747.633- | 3.0154 | 1050.0000 | 4000.0000 | .0000 | .0000 | 1600.0000 | FTTST10401 |

DATA ABSTRACT

OUTPUT LINEAR MODEL
VELOCITY

```
  RANGE      TA-AA     VELOCITY    HEIGHT    ALTITUDE  LATUTUDE   AIM AZ
  METERS     MILS    METERS/SEC OF TARGET OF LNCHR  DEGREES    MILS
 14487602  6-33702000  1 12000000  4 50000000  4 00000000    00000000    DATA
 16000000  4 14575038  6-33881000  1 12000000  4 40000000  4 00000000    DATA
VW=  10  VN=   7  PR. RES.>= 00000000     MAX. TERMS=  96
CON.VALUE= .95000000  STOP ERMS= 45000000-01  TOL.= .00100000
CURR.ERMS=  17266259  3    ADD  TERM   1=R21
CURR.ERMS=  16231338  2    ADD  TERM   2=R22
CURR.ERMS=  88357901  1    ADD  TERM   7=R31
CURR.ERMS=  32506088  1    ADD  TERM   3=R23
CURR.ERMS=  25556902  1    ADD  TERM   8=R31*R21
CURR.ERMS=  18109511  1    ADD  TERM  65=R12
CURR.ERMS=  12018057  1    ADD  TERM  70=R16
CURR.ERMS=  82284316       ADD  TERM   9=R31*R22
CURR.ERMS=  67457624       ADD  TERM   4=R24
CURR.ERMS=  52590605       ADD  TERM  71=R21*R16
CURR.ERMS=  34939190       ADD  TERM  35=R41
CURR.ERMS=  29551889       ADD  TERM  66=R21*R12
CURR.ERMS=  24318115       ADD  TERM  12=R31*R25
CURR.ERMS=  18207841       ADD  TERM  16=R32*R22
CURR.ERMS=  14544096       ADD  TERM   5=R25
CURR.ERMS=  11246925       ADD  TERM  17=R32*R23
CURR.ERMS=  89995059-01    ADD  TERM  36=R41*R21
CURR.ERMS=  77854131-01    ADD  TERM  14=R32
CURR.ERMS=  63666970-01    ADD  TERM  11=R31*R24
CURR.ERMS=  50589079-01    ADD  TERM  79=R31*R16
CURR.ERMS=  47621115-01    ADD  TERM  67=R22*R12
T1= 16400000  1  T2= 16400000  1  TA= 00000000     TR= 00000000

R1=(2*V1-190000)/110000,R3=(V3-925)/225,R4=(V4-2500)/2500,
R5=(V5-1500)/1500,R6=160*V6/9,R7=(V2+V7),R11=C(R6),R12=C(2*R6),
R14=S(R7),R16=R11*R14,
R21=R1,R22=R21*R21,R23=R21*R22,R24=R22**2,R25=R22*R23,R26=R23**2,
R31=R4,R32=R31**2,R33=R31*R32,R34=R32**2,
R41=R5,R42=R41**2,
R3=1+       R21         +R22        +R23        +R24        +R25
+R31        +R31*R21    +R31*R22    +R31*R24    +R31*R25    +R32
+R32*R22    +R32*R23    +R41        +R41*R21    +R12        +R21*R12
+R22*R12    +R16        +R21*R16    +R31*R16    %
COEFFS.                                         NO. OF INPUT LINES=        1440
 21012091     12263420  1-16859641     43141362-01-22583760-01 11779810-01
 49374070-01-17126644-01 66182190-02 33098370-02-45969836-02 75864887-03
 92935093-03-16740335-02 11587382-02 44623851-03-60495567-02-17932391-02
 31790885-03-10469789-01-55085303-02-25963652-03 COEFFS.

NO RESIDUALS               19760329-03 =ERMS       44460740-01 =RS.ERMS
SIGMAS
 14920972-04 41557737-04 67368305-04 16112364-03 70018861-04 13872324-03
 16018734-04 23775661-04 91584837-04 10083039-03 42846665-04 20021938-04
 42222655-04 32381080-04 71017640-05 11630989-04 11074577-04 12885289-04
 21942937-04 13484973-04 22091823-04 17783837-04 SIGMAS
T'S
 14082253  5 29509354  5-25026072  4 26775315  3-32253824  3 84915908  2
 30822704  4-72034352  3 72263261  2 32825788  2-10728918  3 37890881  2
 22007069  2-51697890  2 16316203  3 38366341  2-54625622  3-13916949  3
 14487981  2-77640417  3-24934702  3-14599578  2 T'S
```

CANDIDATE LINEAR MODEL
TARGET AZIMUTH- AIMING AZIMUTH

```
AH=2
NO RESIDUALS
STOP ERMS=.07
R1=(2*V1-190000)/110000,R2=(V2-5)/5,R4=(V4-2500)/2500,
R5=(V5-1500)/1500,R6=160*V6/9,R7=(V2+V7),R10=S(R6),R11=C(R6),
R15=C(R7),R17=R11*R15,
R21=R1,R22=R21*R21,R23=R21*R22,R24=R22**2,R25=R22*R23,R26=R23**2,
R31=R4,R32=R31**2,R33=R31*R32,R34=R32**2,
R41=R5,R42=R41**2,
R2=R21+R22+R23+R24
+R31+R31*R21+R31*R22+R31*R23+R31*R24
+R32+R32*R21+R32*R22+R32*R23
+R33+R33*R21+R33*R22
+R41+R41*R21+R41*R22+R41*R23
+R42+R42*R21+R42*R22+R42*R23
+R41*R31+R41*R31*R21+R41*R31*R22+R41*R31*R23
+R10+R21*R10+R22*R10+R23*R10+R24*R10+R25*R10+R26*R10
+R31*R10+R31*R21*R10+R31*R22*R10+R31*R23*R10+R31*24*R10
+R32*R10+R32*R21*R10+R32*R22*R10+R32*R23*R10+R32*24*R10
+R33*R10+R33*R21*R10+R33*R22*R10+R33*R23*R10
+R34*R10+R34*R21*R10+R34*R22*R10
+R41*R10+R41*R21*R10+R41*R22*R10+R41*R23*R10
+R42*R10+R42*R21*R10+R42*R22*R10+R42*R23*R10
+R41*R31*R10+R41*R31*R21*R10+R41*R31*R22*R10
+R17+R21*R17+R22*R17+R23*R17+R24*R17+R25*R17+R26*R17
+R31*R17+R31*R21*R17+R31*R22*R17+R31*R23*R17+R31*24*R17
+R32*R17+R32*R21*R17+R32*R22*R17+R32*R23*R17+R32*24*R17
+R33*R17+R33*R21*R17+R33*R22*R17+R33*R23*R17
+R34*R17+R34*R21*R17+R34*R22*R17
+R41*R17+R41*R21*R17+R41*R22*R17+R41*R23*R17
+R42*R17+R42*R21*R17+R42*R22*R17+R42*R23*R17
+R41*R31*R17+R41*R31*R21*R17+R41*R31*R22*R17%
```

```
                    OUTPUT LINEAR MODEL
                TARGET AZIMUTH - AIMING AZIMUTH

  RANGE      TA-AA     VELOCITY   HEIGHT   ALTITUDE  LATUTUDE  AIM AZ
  METERS     MILS    METERS/SEC OF TARGET OF LNCHR  DEGREES   MILS
 14487602  6-33702000  1 12000000  4 50000000  4 00000000   00000000   DATA
 16000000  4 14575038  6-33881000  1 12000000  4 40000000  4 00000000   DATA
VW=  10  VN=   7  PR. RES.>= 00000000      MAX. TERMS=  96
CON.VALUE= .95000000  STOP ERMS= 70000000-01  TOL.= .00100000
CURR.ERMS=  55515365  1     ADD  TERM  29=R10
CURR.ERMS=  21252112  1     ADD  TERM  64=R17
CURR.ERMS=  11199558  1     ADD  TERM  30=R21*R10
CURR.ERMS=  87502630        ADD  TERM   1=R21
CURR.ERMS=  41916373        ADD  TERM  65=R21*R17
CURR.ERMS=  24273396        ADD  TERM  17=R41
CURR.ERMS=  20180014        ADD  TERM   3=R23
CURR.ERMS=  17410327        ADD  TERM  18=R41*R21
CURR.ERMS=  14628091        ADD  TERM   5=R31
CURR.ERMS=  12000935        ADD  TERM  31=R22*R10
CURR.ERMS=  10576053        ADD  TERM   2=R22
CURR.ERMS=  90378145-01     ADD  TERM  40=R31*24*R10
CURR.ERMS=  86729010-01     ADD  TERM  71=R31*R17
CURR.ERMS=  80035330-01   REMOVE TERM   5=R31
CURR.ERMS=  80052135-01     ADD  TERM  68=R24*R17
CURR.ERMS=  76440395-01     ADD  TERM  37=R31*R21*R10
CURR.ERMS=  73396210-01     ADD  TERM  20=R41*R23
CURR.ERMS=  71624150-01     ADD  TERM  32=R23*R10
CURR.ERMS=  70405120-01     ADD  TERM  34=R25*R10
T1= 16400000  1  T2= 16400000  1  TA= 00000000     TR= 00000000

R1=(2*V1-190000)/110000,R2=(V2-5)/5,R4=(V4-2500)/2500,
R5=(V5-1500)/1500,R6=160*V6/9,R7=(V2+V7),R10=S(R6),R11=C(R6),
R15=C(R7),R17=R11*R15,
R21=R1,R22=R21*R21,R23=R21*R22,R24=R22**2,R25=R22*R23,R26=R23**2,
R31=R4,R32=R31**2,R33=R31*R32,R34=R32**2,
R41=R5,R42=R41**2,
R2=1+       R21          +R22         +R23         +R41         +R41*R21
+R41*R23    +R10         +R21*R10     +R22*R10     +R23*R10     +R25*R10
+R31*R21*R10+R31*24*R10  +R17         +R21*R17     +R24*R17     +R31*R17%
COEFFS.                                            NO. OF INPUT LINES=        1440
-15701317  1-30252468    32562728-01 92417613-01-40684951-01-58910383-01
 25745388-01 22735780  1 64865455    -10246879    26206386    -19048356
 17669406-01-12805163-02-94902727    -28252271    36217711-01-26415256-01 COEFFS.

NO RESIDUALS              13111114-01 =ERMS        65555570-01 =RS.ERMS
SIGMAS
 10642752-02 24281148-02 21373446-02 32452479-02 47124543-03 19674411-02
 27733189-02 13967718-02 50108064-02 28895955-02 15889529-01 12862190-01
 13457947-02 35664941-04 10956727-02 14657639-02 27692173-02 96661329-03 SIGMAS
T'S
-14753061  4-12459241  3 15235132  2 28477828  2-86334952  2-29942641  2
 92832411  1 16277376  4 12945113  3-35461292  2 16492865  2-14809574  2
 13129347  2-35904062  2-86615949  3-19274775  3 13078681  2-27327636  2 T'S
```

## METEOROLOGY AND CLIMATOLOGY

It was stated earlier that a design objective in developing guided missiles was the creation of a missile which was relatively unaffected by variations in the meteorological conditions of the atmosphere. This is, of course, an idealistic goal seldom, if ever, met in practice. The reason for considering meteorology as a variable in the firing table problem would be to increase the system accuracy. This increase in system accuracy and implied increase in system effectiveness must be weighed against the attendant complications that enter when meteorology is treated as a variable. For example, the question arises as to the requirement for an MOS (Military Occupational Specialty) pertaining to meteorology to be included in the TOE (Table of Organization and Equipment) for the missile battalion. The possible need for additional communication channels between meteorology teams and the missile battalion must also be considered.

When all these factors are considered, a possible compromise solution might be the ignoring of day to day variations in meteorology but the introduction of compensations for seasonal and geographical variations. This has led to the concept of climatology as was used, for example, on the Pershing system. In this scheme certain areas of the globe and certain seasons are categorized into certain climatological groupings. The firing table problem then must concern itself with the incorporation of climatological variation in the computation of the presettings. One solution would be to have a set of fits for each climatological grouping. This implies that the data from the theoretical trajectories must be computed on the basis of a theoretical atmospheric model corresponding to the particular climatological group being studied.

Since the effects of climatology are expected to produce only small changes on the presettings, it might be simpler to introduce corrections to a basic set of fits to compensate for climatology. In this scheme, each climatological group would have a correction fit. Each missile battalion would be given a set of rules indicating which climatological grouping to use under certain geographical and seasonal conditions.

A simple computational scheme can be devised for computing these corrections. The major contributing factor to climatological bias is the density. This density bias affects the trajectory through the retardation or drag and through the interaction effects of the guidance and control electronics caused by the bias on the sensed accelerations. It can be safely assumed that, if these corrections are allowed to be functions of (R, H, A), then most of the error can be corrected. For the computation of a correction fit, two sets of trajectories are required. One set should be computed for standard conditions of the atmosphere. This standard trajectory data can then be used for all ensuing work on other climatological groupings.

In the absence of the Stepwise Multiple Regression Technique, it would be necessary to tabulate the data for equal values of (R, H, A). This would either require iteration of the trajectories (which, as pointed out previously, can be quite expensive) or interpolation of the final data. The difference in the presettings corresponding to particular values of (R, H, A) could then be fitted in terms of polynomials in (R, H, A).

To obtain the correction fits for climatology in one operation by use of Stepwise Multiple Regression, it is necessary to introduce a dummy variable for climatology. This dummy variable, $\xi_c$, takes the value 0 for the standard trajectory data and 1 for the perturbed climatological data. The candidate model for finding the correction on V would be written as

$$V = V_{STD} + \xi_c \Delta V = V_{STD}(R, H, A) + \xi_c \Delta V(R, H, A).$$

All the components of the polynomomial model $\Delta V$ must be multiplied by $\xi_c$. In the output model, $V_{STD}$ is discarded and the significant components of $\Delta V$ are the climatological corrections. A similar procedure can be utilized for finding correction functions for any pre-setting which is affected by climatology.

## HIGHER HARMONICS

It might be suspected that, as the range increases, the appearance of the higher harmonic terms could be explained by an examination of the terms previously neglected in the matrix of minor factors in Equation (10). It usually turns out that the significant harmonic terms which appear next are $\sin 2L \cos A_t$ for range and time and $\sin 2L \sin A_t$ for azimuth. These terms do not appear in the matrix. The origin of these terms could be any or all of the following:

1. The gravitational field for an ellipsoidal earth model has a significant vector component which varies as $\sin 2L$. This component is perpendicular to the geocentric line and tends to pull a missile toward the equator. The variation with $A_t$ is a consequence of this component having a fixed direction, hence, its resolution in the downrange direction would be sinusoidal in $A_t$.

2. The earth's centrifugal force.

3. A second-order interaction effect of two first order harmonic terms.

Regardless of the origin of this term, it is clear that a linear model including higher harmonic terms is required in order to achieve the necessary accuracies for long range systems. The approach presented earlier seems to be the safest and most straightforward way to identify the significant trigonometric terms. Baker and Dinjar[2] list results for a sample exercise with data which are approximated by the Truncated Fourier Series Technique. The data consist of values of $(R, L, A_a, \Delta A, T)$ for $(H, A) = 0$ and for constant V. Approximations are obtained for R (Range angle), $\Delta A$, and T in terms of $(L, A_t)$. The data for range angle were approximated by Stepwise Multiple Regression, and the results are listed for comparison in the following table.

| | Fourier Series Coefficient | Multiple Regression Coefficient |
|---|---|---|
| Const | 6.04386 | 6.04342 |
| $\cos 2L$ | .01194 | .01148 |
| $\cos L \sin A_t$ | .11601 | .11632 |
| $\cos 3L \sin A_t$ | .00035 | ** |
| $\cos 2 A_t$ | -.00092 | ** |
| $\cos 2L \cos A_t$ | -.00093 | ** |
| $\sin 2L \cos A_t$ | -.01286 | -.00961 |
| $\sin 2L \sin 2 A_t$ | -.00023 | ** |
| $\cos L \cos 2 A_t$ | * | -.00237 |
| $\cos 3L \cos 2 A_t$ | * | -.00458 |

*Not included by Baker and Dinjar

**Not present in approximation by multiple regression

An inspection of the results indicates that the coefficients for the most significant harmonics seem to agree, but the difference is observed to be of the same order of magnitude as the least significant harmonics. The least significant harmonics differ not only in the coefficients but also in the terms themselves, i.e., the form of the functions are different. A plausible explanation for these results is the following. The Fourier Series technique (the twelve ordinate method) requires data at 30° intervals. If the data are tabulated for L in the Northern Hemisphere only, an extrapolation process is required to fill in the remaining data. Baker and Dinjar extrapolate their data on the assumption that the data behave as one of the curves (1) sin L, (2) sin 2L, (3) cos L, (4) cos 2L. Since the data do not actually behave as any single one of these functions nor, for that matter as a linear combination of all these functions, the extrapolated data are distorted. One of the derivations for the closed form expressions of the Fourier Series coefficients is based on a least-squares criteria, and, hence, these approximations have properties analogous to least squares, i.e., the coefficients are a sort of compromise. The distortion is now distributed by the mechanics of the fitting process even to those regions of the independent variables for which no extrapolation was done. Even if the resulting model is refitted by least squares, the results are not expected to be the same because the distortion of the data has created a different model than that which would be found by Stepwise Regression analysis on the undistorted data.

This leads to the conclusion that some of the least significant harmonics, when determined by numerical Fourier Series, add nothing to the accuracy of the fit unless the model is refitted by least-squares. Furthermore, since the regression analysis is done on original data only, the terms predicted to be significant by this process should be the most reliable.

To this point, two approaches to the fitting problem have been considered. The first approach is the obtaining of formulas of the type represented by (1). The second approach leads to formulas of the type represented by (5). The Stepwise Multiple Regression procedure can be used to find either type of formula whereas previous procedures seem limited to formulas such as (5). One reservation exists for using Stepwise Multiple Regression with formulas such as (1). As outlined, the first step of the procedure would find the significant harmonics needed to approximate R when V is fixed at its maximum value. The fact that V can be approximated by the same linear combination of terms in $(L, A_t)$, as was required to approximate R, is a conjecture. If, for a given missile system, this conjecture proves false, one has to make a decision whether the relative value to be gained by having formulas such as (1) as compared to (5) is worth the price of further analysis. With a limitation of 100 terms, it might be difficult to choose a good candidate linear model if more than 2 or 3 harmonic terms are significant. The BRL program for the BRLESC computer can be expanded, however, and no real problem should exist even for very long ranges. When the number of terms becomes a problem, the following procedure should prove satisfactory. Suppose a formula of the following type is being sought:

$$R = R_0\,(V, H, A) + R_1\,(V, H, A) \cos 2L + R_2\,(V, H, R) \cos L \sin A_t$$
$$+ R_3\,(V, H, A) \cos L \cos 2 A_t + R_4\,(V, H, A) \cos 3L \cos 2 A_t$$

The relative magnitudes of the terms are known and the complexity of the component candidates for $R_0$, $R_1$, etc. should be chosen accordingly. The components $R_3$ and $R_4$, for example, are quite small relatively and probably can be represented as functions of R only, i. e., they have no significant dependence on (H, A), and their dependence on R is probably no more than quadratic. The term $R_0$, is the most significant and, hence, requires the most comprehensive model in the variables (R, H, A). Therefore, it might be desirable as a preliminary investigation to select data with (L, $A_a$) constant and fit $R = R_0\,(V, H, A)$. $R_0$ might consist of 20-30 terms at most, and thereafter the remaining allocation of terms can be distributed among $R_1$, $R_2$, etc. when forming the complete candidate.

## CONCLUSIONS

The procedures described in this report have been used successfully in the preliminary computations for solving the firing table problem for the Lance missile system. In addition, the sample problem included in this report shows the relative ease with which this type problem can be solved once the fitting program is available. The major advantage of the procedure seems to be that the manipulation of data in several stages is not required. This tends to reduce the effort on the part of the analyst and avoids the possibilities of error at intermediate stages. The discussion has been limited to guided missiles, but the procedure is adaptable to other types of fire control problems for which data must be compressed into compact form.

## Authors Note

Since the completion of this manuscript the author has been successful in modifying the algorithm for Stepwise Multiple Regression so that only half the storage of the original algorithm is now required. This modification now permits the use of candidate models containing 200 terms in the BRLESC program.

HAROLD J. BREAUX

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13 ABSTRACT

The computation of firing tables for guided missiles is a problem that first arose when the U. S. Army introduced the Redstone Missile into its arsenal of weapons. The repetitive nature of such computations, their continuing requirement, and the turnover in personnel create a need for systemization of computations, applicable for different missile systems and at the same time, not requiring extensive analysis, new computer programs, and training of personnel. Recent work done by the Computing Laboratory of BRL accomplishes this objective and is a significant improvement in the state of the art. This report describes the procedure and includes a realistic example of a typical computational problem.

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| Equations of Motion | | | | | | |

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